

# Trigonometry

Trigonometry that makes sense: sine, cosine and tangent in right triangles and on the unit circle, with special angles and exam-style questions.

In a right triangle, once you fix an acute angle  $\theta$ , the **ratios between the sides** are fixed, no matter how big the triangle is. These ratios have names: **sine**, **cosine** and **tangent**. With them, one angle and one side are enough to find the other sides: heights of buildings, ramps, distances you can't measure with a tape.

## Rule 1

### SOH-CAH-TOA

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$$

"Opposite" and "adjacent" depend on which angle you pick. The hypotenuse is always the side across from the right angle. This works **only** in right triangles.

## Rule 2

### Special angles

$$\sin 30^\circ = \frac{1}{2} \quad \sin 45^\circ = \frac{\sqrt{2}}{2} \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$180^\circ = \pi \text{ rad}$$

Cosine is the same list in reverse:

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 45^\circ = \frac{\sqrt{2}}{2}, \cos 60^\circ = \frac{1}{2}. \text{ And } \tan 45^\circ = 1.$$

## Rule 3

### Key identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin \theta = \cos(90^\circ - \theta)$$

The first is the Pythagorean theorem on a circle of radius 1. The second says the two acute angles of a right triangle swap sine and cosine.

## Worked examples, step by step

### Worked example: The accessibility ramp

A ramp makes a  $30^\circ$  angle with the ground and is 12 m long. How high does it rise, and how much floor does it take up (horizontal length)?

- $h = 12 \cdot \frac{1}{2} = 6 \text{ m}.$
- $d = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3} \approx 10.4 \text{ m}.$
- Check with Pythagoras:  $6^2 + (6\sqrt{3})^2 = 36 + 108 = 144 = 12^2 \checkmark.$

**Answer: Height 6 m; floor length  $6\sqrt{3} \approx 10.4 \text{ m}$**

### Worked example: Height of a building without climbing it

From a point on the ground 20 m from the base of a building, a person sees the top at a  $60^\circ$  angle of elevation. How tall is the building? (Use  $\sqrt{3} \approx 1.73$ .)

- $\tan 60^\circ = \frac{h}{20}.$
- $h = 20 \cdot \sqrt{3}.$
- $h \approx 20 \cdot 1.73 = 34.6 \text{ m}.$

**Answer:  $20\sqrt{3} \approx 34.6 \text{ m}$**

## Mistakes that cost points

### Swapping opposite and adjacent

#### ✗ Wrong

3-4-5 triangle, opposite  $\theta$  is 3:  
 $\cos \theta = \frac{3}{5}$

#### ✓ Right

Cosine uses the **adjacent** side:  $\cos \theta = \frac{4}{5}$ . Mark on the drawing which side touches the angle.

### Calculator in radian mode

#### ✗ Wrong

On the calculator:  $\sin(30) = -0.988$

#### ✓ Right

The calculator was in radians (RAD). In degrees (DEG),  $\sin 30^\circ = 0.5$ .

### Thinking sine is proportional to the angle

#### ✗ Wrong

$$\sin 60^\circ = 2 \cdot \sin 30^\circ = 1$$

#### ✓ Right

Sine does not grow in a straight line:  $\sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0.87$ .

### Misreading $\sin^2 \theta$

#### ✗ Wrong

$$\sin^2 \theta = \sin(\theta^2)$$

#### ✓ Right

$\sin^2 \theta = (\sin \theta)^2$ : take the sine first, then square it.