

Powers and roots

Exponents and roots without memorizing: understand the rules, zero and negative exponents, scientific notation and how to simplify roots.

An exponent is repeated multiplication: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. Every rule comes from that: **count how many equal factors there are**. A root is the inverse operation: $\sqrt{9} = 3$ because $3^2 = 9$. That is why a root can also be written as a fractional power: $\sqrt{a} = a^{1/2}$.

Rule 1

The rules

$$a^m \cdot a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \cdot n} \quad (ab)^n = a^n b^n$$

Same base: when multiplying, **add** the exponents; when dividing, **subtract**. Power of a power: **multiply**.

Rule 2

Zero and negative exponents

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

A negative exponent does **not** make the number negative: it "flips" it. $2^{-3} = \frac{1}{8}$ and $(\frac{2}{3})^{-1} = \frac{3}{2}$.

Rule 3

Roots

$$\sqrt[n]{a^m} = a^{m/n} \quad \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

To simplify, look for a perfect square inside the root: $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$. But careful: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.

Worked examples, step by step

Worked example: Scientific notation in practice

The Earth is about 150,000,000 km from the Sun, and light travels 300,000 km per second. How long does sunlight take to reach the Earth?

- $t = \frac{1.5 \times 10^8}{3 \times 10^5}$.
- $\frac{1.5}{3} = 0.5$ and $\frac{10^8}{10^5} = 10^3$, so $t = 0.5 \times 10^3 = 500$ s.
- 500 s = 8 min 20 s.

Answer: 500 s, about 8 minutes

Worked example: Simplifying and adding roots

Compute $\sqrt{50} + \sqrt{18} - \sqrt{8}$.

- $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$; $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$; $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$.
- $5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = (5 + 3 - 2)\sqrt{2}$.
- $= 6\sqrt{2} \approx 8.49$.

Answer: $6\sqrt{2}$

Mistakes that cost points

Multiplying exponents in a product

✗ Wrong

$$2^3 \cdot 2^4 = 2^{12}$$

✓ Right

Count the factors: $3 + 4 = 7$. $2^3 \cdot 2^4 = 2^7$. Exponents multiply only for a power of a power.

Thinking a negative exponent gives a negative number

✗ Wrong

$$2^{-3} = -8$$

✓ Right

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$: positive and small.

Mixing up -2^4 and $(-2)^4$

✗ Wrong

$$-2^4 = 16$$

✓ Right

Without parentheses the exponent applies only to the 2: $-2^4 = -(2^4) = -16$. But $(-2)^4 = 16$.

Splitting the root of a sum

✗ Wrong

$$\sqrt{9+16} = 3+4 = 7$$

✓ Right

Add first: $\sqrt{25} = 5$. A root only splits over **products**: $\sqrt{9 \cdot 16} = 3 \cdot 4$.