

Percentage and ratio

Percentages and proportions for real life: discounts, successive changes, ratios and the traps that show up most on exams.

A percentage is just a fraction out of 100: $35\% = \frac{35}{100} = 0.35$. The exam secret is to think in **multipliers**: increasing by 20% means multiplying by 1.2; a 20% discount means multiplying by 0.8. With that, successive changes, discounts and “what was the original price?” all become a single calculation.

Rule 1

Percent of a number

$$p\% \text{ of } N = \frac{p}{100} \cdot N$$

Mental shortcuts: 10% means moving the decimal point one place; 5% is half of 10%; 1% is dividing by 100.

Rule 2

Multipliers

$$p\% \text{ increase} : \times \left(1 + \frac{p}{100}\right)$$

$$p\% \text{ decrease} : \times \left(1 - \frac{p}{100}\right)$$

Successive changes **multiply** the factors, they do not add the percentages: +20% then +20% gives $1.2 \cdot 1.2 = 1.44$, that is, +44%.

Rule 3

Proportion

$$\text{direct: } \frac{a}{b} = \frac{c}{d} \quad \text{inverse: } a \cdot b = c \cdot d$$

Direct: if one doubles, so does the other (price and quantity). Inverse: if one doubles, the other halves (speed and time).

Worked examples, step by step

Worked example: Successive changes

A stock rises 30% one month and falls 30% the next. Did someone who bought at the start gain or lose? By what percent?

1. Total multiplier: $1.3 \cdot 0.7 = 0.91$.
2. With \$100: it rises to 130, then falls 30% of 130 (39) to 91.
3. 0.91 is 9% below 1: a 9% loss.

Answer: A 9% loss

Worked example: Finding the original price

After a 20% discount, a shirt costs \$64. What was the price before the discount?

1. Equation: $0.8p = 64$.
2. $p = \frac{64}{0.8} = 80$.
3. Check: 20% of 80 is 16 and $80 - 16 = 64$ ✓.

Answer: \$80

Mistakes that cost points

Adding successive percentages

✗ **Wrong**

+20% then +20% = +40%

✓ **Right**

Multiply the multipliers: $1.2 \cdot 1.2 = 1.44$, that is, +44%.

Thinking equal increase and decrease cancel out

✗ **Wrong**

+10% then -10%: back to the starting price

✓ **Right**

$1.1 \cdot 0.9 = 0.99$: it ends 1% lower. The discount is taken from a bigger amount.

Undoing a discount by adding the percentage

✗ **Wrong**

Costs 64 after a 20% discount: original = $64 + 20\% \cdot 64 = 76.80$

✓ **Right**

The discount was on the original price: $0.8p = 64 \Rightarrow p = 80$.

Treating an inverse proportion as direct

✗ **Wrong**

6 painters take 10 days $\Rightarrow$ 12 painters take 20 days

✓ **Right**

More painters, **fewer** days: $6 \cdot 10 = 12 \cdot d \Rightarrow d = 5$ days.