

Intro to algebra

Intro to algebra: understand what an equation is, solve it step by step like a balance scale and avoid the sign errors that cost points.

Algebra is arithmetic with an open question. In $3 + x = 8$, the letter x holds a number we do not know yet. Solving means finding the value that makes the sentence true **without guessing**: by doing **the same operation to both sides**, like a balance scale that must stay level.

Rule 1

The balance

$$a = b \Rightarrow a + c = b + c$$

$$a = b \Rightarrow a \cdot c = b \cdot c$$

Add, subtract, multiply or divide both sides by the same number (divide only by a nonzero number).

Rule 2

Distributive property

$$a(b + c) = ab + ac$$

The number outside multiplies **every** term inside. With a minus sign: $-(x - 4) = -x + 4$.

Rule 3

Like terms

$$3x + 5x = 8x \quad 2x + 3 \neq 5x$$

Only terms with the same letter and the same exponent combine. Plain numbers combine with plain numbers.

Worked examples, step by step

Worked example: The unknown on both sides

Solve $7x - 4 = 3x + 20$.

1. Subtract $3x$ from both sides: $4x - 4 = 20$.
2. Add 4 to both sides: $4x = 24$.
3. Divide by 4: $x = 6$. Check: $7 \cdot 6 - 4 = 38$ and $3 \cdot 6 + 20 = 38 \checkmark$.

Answer: $x = 6$

Worked example: From words to an equation

A number plus three times that number is 48. What is the number?

1. Equation: $x + 3x = 48$.
2. Combine like terms: $4x = 48$.
3. Divide by 4: $x = 12$. Check: $12 + 36 = 48 \checkmark$.

Answer: 12

Mistakes that cost points

Distributing to only one term

✗ Wrong

$$3(x + 4) = 3x + 4$$

✓ Right

The 3 multiplies **both** terms: $3x + 12$.

The minus sign before parentheses

✗ Wrong

$$-(x - 4) = -x - 4$$

✓ Right

The minus flips the sign of **every** term: $-(x - 4) = -x + 4$.

Substituting a negative number without parentheses

✗ Wrong

$$\text{For } a = -2: a^2 = -2^2 = -4$$

✓ Right

Always use parentheses: $a^2 = (-2)^2 = 4$.

Combining unlike terms

✗ Wrong

$$2x + 3 = 5x$$

✓ Right

$2x$ and 3 are not like terms: $2x + 3$ is already simplified.