

Plane geometry

Plane geometry that makes sense: angles, areas and the Pythagorean theorem explained with pictures, exam examples and a tool to try.

Almost every plane-geometry question uses just three ideas: **the angles of a triangle add up to 180°** , **area is how many 1×1 squares fit inside the shape** and, in every right triangle, $a^2 + b^2 = c^2$. Master these three and you can solve most problems by drawing the figure and writing one equation.

Rule 1

Angle sum

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

$$S_n = (n - 2) \cdot 180^\circ$$

In a polygon with n sides, the interior angles add up to $(n - 2) \cdot 180^\circ$: it splits into $n - 2$ triangles.

Rule 2

Key areas

$$A_{\square} = b \cdot h \quad A_{\triangle} = \frac{b \cdot h}{2}$$

$$A_{\circ} = \pi r^2$$

The height h is always **perpendicular** to the base. A triangle is half of a rectangle with the same base and height.

Rule 3

Pythagorean theorem

$$a^2 + b^2 = c^2$$

Right triangles only. c is the hypotenuse, the side opposite the 90° angle (the longest side). Famous triples: 3-4-5, 5-12-13, 8-15-17.

Worked examples, step by step

Worked example: Area of a composite shape

A plot of land is a 10 m by 6 m rectangle with a 4 m by 3 m rectangular corner removed (making an "L"). What are the area and the perimeter of the plot?

1. Area of the whole rectangle: $10 \cdot 6 = 60 \text{ m}^2$.
2. Area of the corner: $4 \cdot 3 = 12 \text{ m}^2$. Area of the L: $60 - 12 = 48 \text{ m}^2$.
3. Perimeter: sides 10, 3, 4, 3, 6, 6... going all the way around gives $2 \cdot (10 + 6) = 32 \text{ m}$, the same as the rectangle.

Answer: Area 48 m^2 , perimeter 32 m

Worked example: The Pythagorean theorem in real life

A 10 m ladder leans against a wall with its foot 6 m from the wall. How high does it reach? If the foot is pulled 2 m further away, how far does the top slide down?

1. $h^2 = 100 - 36 = 64$, so $h = 8 \text{ m}$.
2. With the foot 8 m away: $h^2 = 100 - 64 = 36$, so $h = 6 \text{ m}$.
3. The top slides down $8 - 6 = 2 \text{ m}$.

Answer: 8 m high; the top slides down 2 m

Mistakes that cost points

Forgetting to divide by 2 for a triangle

✗ Wrong

Triangle with base 10 and height 6: area = 60

✓ Right

$A = \frac{b \cdot h}{2} = \frac{60}{2} = 30$. The triangle is half of the rectangle.

Using the slanted side as the height

✗ Wrong

Parallelogram with sides 8 and 5 (height 4): area = $8 \cdot 5 = 40$

✓ Right

The height is the **perpendicular** distance between the bases: $A = 8 \cdot 4 = 32$.

Adding the legs to get the hypotenuse

✗ Wrong

Legs 3 and 4: hypotenuse = $3 + 4 = 7$

✓ Right

$c = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Add the **squares**, then take the square root.

Thinking that doubling the side doubles the area

✗ Wrong

Square with side 3 (area 9) → side 6: area 18

✓ Right

Area = $6^2 = 36$, that is, 4 times larger. Scaling by k multiplies the area by k^2 .