

Functions (linear, quadratic & more)

Linear and quadratic functions that make sense: slope, intercepts, the quadratic formula and the vertex, explained with interactive graphs and exam-style questions.

A function is a rule that turns each input x into an output y . In a **linear function** ($y = ax + b$) the output changes at a constant rate: the graph is a **line**, and a is how much y rises when x increases by 1. In a **quadratic function** ($y = ax^2 + bx + c$) the rate changes: the graph is a **parabola**, with a highest or lowest point, the **vertex**.

Rule 1

Linear function

$$f(x) = ax + b \quad a = \frac{y_2 - y_1}{x_2 - x_1}$$

a is the slope (rate of change); b is where the line crosses the y -axis. The zero (where it crosses the x -axis) is $x = -\frac{b}{a}$.

Rule 2

Quadratic zeros

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \quad \Delta = b^2 - 4ac$$

$\Delta > 0$: two real zeros; $\Delta = 0$: one (double) zero; $\Delta < 0$: no real zeros. Shortcut: the zeros add to $-\frac{b}{a}$ and multiply to $\frac{c}{a}$.

Rule 3

Vertex

$$x_v = -\frac{b}{2a} \quad y_v = f(x_v) = -\frac{\Delta}{4a}$$

$a > 0$: the parabola opens up and the vertex is the **minimum**. $a < 0$: it opens down and the vertex is the **maximum**. x_v is halfway between the two zeros.

Worked examples, step by step

Worked example: When one plan becomes cheaper
Gym A charges a \$100 sign-up fee and \$80 per month. Gym B has no sign-up fee but charges \$100 per month. After how many months is gym A cheaper?

1. Set equal: $100 + 80m = 100m \Rightarrow 100 = 20m \Rightarrow m = 5$.
2. At 5 months both cost \$500.
3. Since A grows 80 per month and B grows 100, from month 6 on A is cheaper (at 6 months: A = 580, B = 600).

Answer: From month 6 on

Worked example: The quadratic formula start to finish
Solve $2x^2 - 3x - 2 = 0$.

1. $\Delta = (-3)^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25$.
2. $x = \frac{3 \pm 5}{4}$.
3. $x_1 = \frac{8}{4} = 2$ and $x_2 = \frac{-2}{4} = -\frac{1}{2}$. Check with the sum:
 $2 - \frac{1}{2} = \frac{3}{2} = -\frac{b}{a} \checkmark$.

Answer: $x = 2$ or $x = -\frac{1}{2}$

Mistakes that cost points

Flipping the slope formula

✗ **Wrong**

From (1, 3) to (3, 7): $a = \frac{3-1}{7-3} = \frac{1}{2}$

✓ **Right**

Slope is change in y over change in x : $\frac{7-3}{3-1} = 2$.

Getting the sign of $-b$ wrong

✗ **Wrong**

$x^2 - 5x + 6 = 0$: $x = \frac{-5 \pm 1}{2}$, zeros -2 and -3

✓ **Right**

$b = -5$, so $-b = +5$: $x = \frac{5 \pm 1}{2}$, zeros 2 and 3.

Forgetting the negative root

✗ **Wrong**

$x^2 = 9 \Rightarrow x = 3$

✓ **Right**

$x = \pm 3$: both 3^2 and $(-3)^2$ equal 9.

Dividing by x and losing the zero root

✗ **Wrong**

$x^2 = 5x \Rightarrow x = 5$

✓ **Right**

Factor instead of dividing:
 $x(x - 5) = 0$, so $x = 0$ or $x = 5$.