

Fractions and decimals

Fractions and decimals without memorizing: understand what a fraction means, do the four operations with confidence and avoid the classic mistakes.

Fractions, decimals and percentages are **three ways to write the same number**: $\frac{3}{4} = 0.75 = 75\%$. And $\frac{a}{b}$ simply means $a \div b$. That is why $\frac{6}{3} = 2$, no pizza drawing required.

Rule 1

Equivalent fractions

$$\frac{a}{b} = \frac{a \cdot k}{b \cdot k}$$

Multiplying or dividing the top and bottom by the same number does not change the fraction. That is how you simplify.

Rule 2

Adding and subtracting

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Only add numerators when the denominators match. First make the denominators equal (use the least common multiple).

Rule 3

Multiplying and dividing

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To multiply, top times top and bottom times bottom. To divide, multiply by the reciprocal of the second fraction.

Worked examples, step by step

Worked example: Adding with different denominators

Find $\frac{5}{6} + \frac{3}{8}$.

1. LCM(6, 8) = 24.
2. $\frac{5}{6} = \frac{20}{24}$ and $\frac{3}{8} = \frac{9}{24}$.
3. $\frac{20}{24} + \frac{9}{24} = \frac{29}{24}$, already in lowest terms (= $1\frac{5}{24}$).

Answer: $\frac{29}{24}$

Worked example: Dividing decimals

A bottle holds 1.5 liters of juice. How many 0.25-liter glasses can it fill?

1. Division: $1.5 \div 0.25$.
2. Multiplying both by 100: $150 \div 25 = 6$.
3. Check: $6 \times 0.25 = 1.5$ ✓. (Or with fractions: $\frac{3}{2} \div \frac{1}{4} = \frac{3}{2} \cdot 4 = 6$.)

Answer: 6 glasses

Mistakes that cost points

Adding tops and bottoms

✗ Wrong

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$$

✓ Right

Make the denominators equal first: $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

Dividing without Flipping

✗ Wrong

$$\frac{2}{3} \div \frac{4}{5} = \frac{8}{15}$$

✓ Right

Multiply by the **reciprocal** of the second: $\frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

The decimal point when multiplying

✗ Wrong

$$0.3 \times 0.2 = 0.6$$

✓ Right

Count the decimal places in both factors: $1 + 1 = 2$ places, so 0.06.

Comparing decimals by their length

✗ Wrong

$0.125 > 0.5$, because 125 is bigger than 5.

✓ Right

Pad with zeros and compare place by place: $0.500 > 0.125$.