

Statistics and probability

Probability without memorizing: favorable outcomes, complements, “and” and “or”, without replacement and conditional probability from tables, with a simulator and exam-style questions.

Probability is a fraction: **favorable outcomes over possible outcomes**, from 0 (impossible) to 1 (certain). The formula is easy; the hard part is **counting correctly**. Three questions solve almost everything: is it easier to count the opposite (complement)? Is it “and” (multiply) or “or” (add, without double counting)? And, in a table, **what is the total** after the “given that”?

Rule 1

Definition and complement

$$P(A) = \frac{\text{favorable}}{\text{possible}}$$

$$P(\text{not } A) = 1 - P(A)$$

This works when all outcomes are equally likely. For “at least one”, it is almost always easier to compute $1 - P(\text{none})$.

Rule 2

“And” and “or”

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

“And” multiplies (for independent events); “or” adds and removes what was counted twice. Without replacement the second probability changes: the total shrinks.

Rule 3

Conditional probability

$$P(A | B) = \frac{n(A \text{ and } B)}{n(B)}$$

“Given that B happened” changes the total: you only look at the cases in B . In a table, read **only the row or column** of the condition.

Worked examples, step by step

Worked example: The two-dice table

Rolling two dice, what is the probability that the sum is 7? And that the sum is at least 10?

- Sum 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), that is, 6 pairs: $\frac{6}{36} = \frac{1}{6}$.
- Sum 10: (4, 6), (5, 5), (6, 4); sum 11: (5, 6), (6, 5); sum 12: (6, 6). Total: 6 pairs.
- $P(\text{sum} \geq 10) = \frac{6}{36} = \frac{1}{6}$. Interesting: the same chance as a sum of 7 alone.

Answer: $\frac{1}{6}$ and $\frac{1}{6}$

Worked example: “At least one” using the complement

A light bulb has a 10% chance of burning out in its first year. With 3 bulbs that work independently, what is the probability that **at least one** burns out in the first year?

- $P(\text{none burns out}) = 0.9 \cdot 0.9 \cdot 0.9 = 0.729$.
- $P(\text{at least one}) = 1 - 0.729 = 0.271$.
- About 27%. Notice it is not $3 \cdot 10\% = 30\%$: adding would double-count cases.

Answer: $0.271 \approx 27\%$

Mistakes that cost points

Adding when it is “and”

× Wrong

$$P(\text{two heads}) = \frac{1}{2} + \frac{1}{2} = 1$$

✓ Right

“And” with independent events multiplies: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. If you got 1 (certainty), be suspicious.

Double counting with “or”

× Wrong

$$P(\text{ace or heart}) = \frac{4}{52} + \frac{13}{52} = \frac{17}{52}$$

✓ Right

The ace of hearts was counted twice: $\frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}$.

Forgetting it is without replacement

× Wrong

3 red and 2 blue, two draws:
 $P = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

✓ Right

Without replacement the box changes: $\frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$.

Using the wrong total in a conditional

× Wrong

$$P(\text{girl} | \text{music}) = \frac{45}{150}$$

✓ Right

“Given music”: the total is the music column, 75. $\frac{45}{75} = \frac{3}{5}$.