

Basic statistics

Basic statistics that makes sense: mean, median, mode and spread explained with real data, so you can read tables and charts without falling for traps.

Statistics sums up many numbers with a few. **Measures of center** (mean, median, mode) say where the data sit; **measures of spread** (range, standard deviation) say how spread out they are. The key exam question is: **which measure best represents this data?** When there is a value far from the rest, the median is usually the answer.

Rule 1

Mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$\bar{x}_p = \frac{p_1 x_1 + \dots + p_n x_n}{p_1 + \dots + p_n}$$

Add everything and divide by how many there are. In a weighted mean, each value counts as many times as its weight. Handy trick: $\text{sum} = \text{mean} \cdot n$.

Rule 2

Median and mode

$$1, 3, \boxed{5}, 7, 9 \quad 2, \boxed{4, 6}, 8 \rightarrow \frac{4+6}{2} =$$

Median: **sort** the data and take the middle value (with an even count, the average of the two middle values). Mode: the most frequent value (there may be several, or none).

Rule 3

Spread

$$\text{range} = \max - \min$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

The standard deviation σ is the "typical distance" from the data to the mean. All values equal: $\sigma = 0$. More spread out: larger σ .

Worked examples, step by step

Worked example: The missing score

To pass, Ana needs a mean of 7 over 4 tests. She scored 6, 8 and 5.5 on the first three. What does she need on the fourth?

1. Sum needed: $7 \cdot 4 = 28$.
2. Current sum: $6 + 8 + 5.5 = 19.5$.
3. Missing: $28 - 19.5 = 8.5$. Check: $\frac{6+8+5.5+8.5}{4} = \frac{28}{4} = 7 \checkmark$.

Answer: 8.5

Worked example: Weighted mean

An entrance exam gives weight 3 to Math, 2 to Language and 1 to the Essay. João scored 8, 6 and 9, respectively. What is his final average?

1. Products: $3 \cdot 8 = 24$, $2 \cdot 6 = 12$, $1 \cdot 9 = 9$. Sum: 45.
2. Sum of the weights: $3 + 2 + 1 = 6$.
3. Average: $\frac{45}{6} = 7.5$. (The simple mean would be $\frac{23}{3} \approx 7.67$: the heavier weight on Math changes the result.)

Answer: 7.5

Mistakes that cost points

Finding the median without sorting

✗ Wrong

3, 9, 1, 7, 5: median = 1 (the middle one)

✓ Right

Sort first: 1, 3, 5, 7, 9. Median = 5.

Trusting the mean with an outlier

✗ Wrong

Salaries 2, 2, 3, 3, 40: "the typical employee earns 10 thousand"

✓ Right

One extreme value pulls the mean. The median (3 thousand) better describes what is typical.

Averaging averages of different-sized groups

✗ Wrong

Class A (10 students) mean 8; class B (30 students) mean 6 $\Rightarrow$ overall mean 7

✓ Right

Weight by size: $\frac{10 \cdot 8 + 30 \cdot 6}{40} = \frac{260}{40} = 6.5$.

The median with an even count

✗ Wrong

2, 4, 6, 8: median = 4 (or 6)

✓ Right

With an even count, average the two middle values: $\frac{4+6}{2} = 5$.